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Translated by M.D.F.

ELASTIC EQUILIBRIUM OF A PLATE WITH A PARTIALLY REINFORCED CURVILINEAR HOLE*

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An approximate method is proposed to determine the state of stress near a curvilinear hole whose outline is partially reinforced by a thin elastic rod of variable cross-section in an infinite plate. The problem is reduced to a system of two singular integral equations in the contact stresses by the method of complex-function theory /1/, and the method of boundary collocation is used to solve them /2/. Certain special cases of the problem and numerical examples are examined.

Problems of reducing the stress concentration around circular holes in plates have been discussed in /3, 4/. In practice, all the previous investigations on this problem have been devoted to problems of reinforcement of the whole hole outline by rods of constant or variable cross-section.

1. We consider an infinite isotropic plate of thickness $2h$ with a circular hole of radius $\rho_0 = 1$. Part of the hole outline, determining the central angle $2\alpha_0$, is reinforced by a thin elastic rod of variable cross-section. We consider the thickness of the reinforcement to be constant, and the width to be a continuous smooth function of the arc. The plate is subjected to bending in two mutually perpendicular directions by the moments $M_1 = M_x^\infty$, $M_2 = M_y^\infty$ applied at "infinity". There is not external load on the hole outline.

The plate middle plane is referred to a ρ, λ polar coordinate system with the pole at the centre of the hole. The polar axis passes through the middle of the reinforcing rod and makes an angle β_0 with the direction of action of the moment M_1. We consider the rod as an elastic line subjected to bending and torsion /3/.

The boundary conditions of the boundary value problem and its solution on the hole outline, in the notation of /1/, have the form

$$\varkappa \Phi^-(t_0) + \Phi^+(t_0) = -kf(t_0) \tag{1.1}$$
$$\Phi^+(t_0) - \Phi^-(t_0) = k(1+\nu)^{-1}[M_\lambda - \nu M_\rho - i(1+\nu)H_{\rho\lambda}]$$

$$\Phi^-(t_0) = \frac{k}{2\varkappa} \left[\frac{1}{\pi i} \int_L \frac{f(\tau)\,d\tau}{\tau - t_0} - f(t_0) \right] + \Gamma -$$

$$\varkappa^{-1}\Gamma' \exp\left[-2i(\lambda + \beta_0)\right], \quad \tau = \exp(it)$$

$$\Phi^+(t_0) = -\frac{k}{2} \left[\frac{1}{\pi i} \int_L \frac{f(\tau)\,d\tau}{\tau - t_0} + f(t_0) \right] - \varkappa\Gamma +$$

$$\Gamma' \exp\left[-2i(\lambda + \beta_0)\right], \quad t_0 = \exp(i\lambda)$$

$$f(t_0) = M_\rho + i\mathbf{P}, \quad \mathbf{P} = H_{\rho\lambda} + \int_0^\lambda N_\rho(s)\,ds, \quad L \equiv [-\alpha_0, \alpha_0]$$

$$k^{-1} = D(1-\nu), \quad \varkappa = \frac{3+\nu}{1-\nu}, \quad \Gamma = -\frac{M_1 + M_2}{4D(1+\nu)}$$

$$\Gamma' = -k\,\frac{M_1 - M_2}{2}$$

Using the relationship /3/

$$M_\lambda - \nu M_\rho = -k^{-1}(1+\nu)X, \quad H_{\rho\lambda} = -k^{-1}Y \tag{1.2}$$

$$X = \frac{1}{\rho}\frac{\partial w}{\partial\rho} + \frac{1}{\rho^2}\frac{\partial^2 w}{\partial\lambda^2}, \quad Y = \frac{1}{\rho}\frac{\partial^2 w}{\partial\rho\,\partial\lambda} - \frac{1}{\rho^2}\frac{\partial w}{\partial\lambda}$$

and (1.1) we find for $\rho = \rho_0$

$$X - iY = \Phi^-(t_0) - \Phi^+(t_0) \tag{1.3}$$

Separating real and imaginary parts in (1.3), we determine on the hole outline

$$X = \frac{k}{3+\nu}\left[I(\lambda) + (1+\nu)M_\rho(\lambda) - X_0\right] \tag{1.4}$$

$$Y = -\frac{k}{3+\nu}\left[J(\lambda) + (1+\nu)\mathbf{P}(\lambda) - Y_0\right]$$

$$X_0 - iY_0 = \frac{3+\nu}{1+\nu}(M_1 + M_2) - 2(M_1 - M_2)\exp\left[2i(\lambda + \beta_0)\right]$$

$$I(\lambda) = \frac{1}{\pi}\int_{-\alpha_0}^{\alpha_0}\left[M_\rho(t) - \mathbf{P}(t)\operatorname{ctg}\frac{\lambda - t}{2}\right]dt, \quad J(\lambda) = \frac{1}{\pi}\int_{-\alpha_0}^{\alpha_0}\left[\mathbf{P}(t) + M_\rho(t)\operatorname{ctg}\frac{\lambda - t}{2}\right]dt$$

We find the annular moment M_λ and torsional moment $H_{\rho\lambda}$ by means of the formulas

$$M_\lambda = \frac{\nu - 1}{3+\nu}M_\rho + \frac{1+\nu}{3+\nu}\left[-I(\lambda) + X_0\right] \tag{1.5}$$

$$H_{\rho\lambda} + \frac{1}{3+\nu}\left[J(\lambda) + (1+\nu)\mathbf{P} - Y_0\right]$$

To deduce the boundary conditions of the problem, we separate the reinforcing rod from the plate by replacing its action by unknown contact moments M_ρ and $\mathbf{P}$ (Fig.1). Then

$$L_\rho = \int_\lambda^{\alpha_0}\left[M_\rho(t)\sin(t - \lambda) + \mathbf{P}(t)\cos(t - \lambda)\right]dt \tag{1.6}$$

$$L_\lambda = \int_\lambda^{\alpha_0}\left[M_\rho(t)\cos(t - \lambda) - \mathbf{P}(t)\sin(t - \lambda)\right]dt, \quad \lambda \in [-\alpha_0, \alpha_0]$$

$$M_\rho = \mathbf{P} = 0, \quad \lambda \in [\alpha_0, 2\pi - \alpha_0]$$

where L_ρ, L_λ are the bending and twisting moments in the rod cross-section.

In the theory of thin curvilinear rods /3/

$$L_\rho = -A(\lambda)X, \quad L_\lambda = -C(\lambda)Y \tag{1.7}$$

($A(\lambda), C(\lambda)$ are the variable bending torsion stiffnesses of the rod). Substituting (1.7) into (1.6), taking (1.4) into account, we obtain a system of two singular integral equations to determine the contact moments M_ρ and $\mathbf{P}$ on the juncture section of the plate and the rod

$$\delta\left[I(\lambda) + (1+\nu)M_\rho(\lambda)\right] - N(\lambda)\sin\lambda + \tag{1.8}$$

$$R(\lambda)\cos\lambda = \delta X_0, \quad \omega_0\delta\left[J(\lambda) + (1+\nu)\mathbf{P}(\lambda)\right] +$$

$$N(\lambda)\cos\lambda + R(\lambda)\sin\lambda = \omega_0\delta Y_0$$

Here

$$N(\lambda) = -\int_\lambda^{\alpha_0}\left[M_\rho(t)\cos t - \mathbf{P}(t)\sin t\right]dt, \quad \omega_0 = \frac{C(\lambda)}{A(\lambda)}$$

$$R(\lambda) = - \int\limits_{\lambda}^{\alpha_0} [M_\rho(t)\sin t + \mathbf{P}(t)\cos t]\,dt, \quad \delta = \frac{3+\nu}{kA(\lambda)}$$

The equilibrium conditions for the rod

$$N(-\alpha_0) = 0, \quad R(-\alpha_0) = 0 \tag{1.9}$$

which are equivalent to the conditions for the plate deformation to be single-valued, should be satisfied in addition to system (1.8).
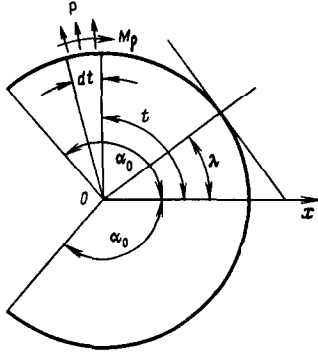

Fig.1

Relations (1.8) are a system of two singular integral equations with Hilbert kernels. Investigation of a system of the kind mentioned in a special case is presented in /2/. If the solution of system (1.8) that satisfies conditions (1.9) is known, then the stresses on the hole outline are determined by (1.5), and in the plate by well-known formulas /1, 5/.

We will consider some special cases of the boundary value problem (1.8), (1.9).

If $A(\lambda) = C(\lambda) = 0$ (no reinforcing rod), then we obtain the solution of the problem for an infinite plate with an unreinforced circular rim. In this case the boundary conditions have the form $M_\rho = 0, \mathbf{P} = 0$. Conditions (1.9) are satisfied identically.

If $A(\lambda) = C(\lambda) = \infty$ (absolutely rigid rod), the system (1.8) takes the form

$$I(\lambda) + (1+\nu)M_\rho(\lambda) = X_0, \quad J(\lambda) + (1+\nu)\mathbf{P}(\nu) = Y_0$$

2. The exact solution of system (1.8) is fraught with great mathematical difficulties. The Multhopp-Kalandiya method of boundary collocation /2/ is applied for its approximate solution.

We will seek the solution of a system unconstrained at the ends in the form

$$M_\rho(\lambda) = \sum_{n=1}^{N_0} A_n F(\varphi, \varphi_n), \quad \mathbf{P}(\lambda) = \sum_{n=1}^{N_0} B_n F(\varphi, \varphi_n) \tag{2.1}$$

$$\lambda = 2\,\mathrm{arctg}\,a_0\cos\varphi, \quad a_0 = \mathrm{tg}\,\frac{\alpha_0}{2}$$

$$F(\varphi, \varphi_n) = \frac{(-1)^{n+1}\cos N_0\varphi\,\sin\varphi_n}{N_0\sin\varphi(\cos\varphi - \cos\varphi_n)}, \quad \varphi_m = \frac{2m-1}{2N_0}\pi$$

(φ_m are Chebyshev nodes for the Lagrange interpolation polynomial, and N_0 is the number of collocation points).

Using (2.1) and the quadrature formulas for singular integrals /2/, we find values of the quantities from (1.8) at the collocation points

$$M_\rho(\lambda_m) = \frac{A_m}{\sin\varphi_m}, \quad \frac{1}{\pi}\int\limits_{-\alpha_0}^{\alpha_0} M_\rho(t)\,dt = \frac{2}{N_0}\sum_{n=1}^{N_0}\frac{A_n}{q_n\sin\varphi_n} \tag{2.2}$$

$$\frac{1}{\pi}\int\limits_{-\alpha_0}^{\alpha_0} M_\rho(t)\,\mathrm{ctg}\,\frac{\lambda_m - t}{2}\,dt = \frac{2}{N_0}\sum_{n+1}^{N_0} A_n\left[\frac{a_0}{q_n\,\mathrm{tg}\,\varphi_n} - \frac{\mathrm{ctg}\,\psi}{\sin\varphi_m}\right]$$

$$\lambda \in [-\alpha_0, \alpha_0]$$

$$\frac{1}{\pi}\int\limits_{-\alpha_0}^{\alpha_0} M_\rho(t)\,\mathrm{ctg}\,\frac{\lambda - t}{2}\,dt = \frac{2}{N_0}\sum_{n=1}^{N_0} A_n\left[\frac{a_0}{q_n\,\mathrm{tg}\,\varphi_n} - \frac{a_0}{a_0\cos\varphi_n - \mathrm{tg}\,\lambda/2}\right], \quad \lambda \in [\alpha_0, 2\pi - \alpha_0]$$

$$\left(q_n = \frac{1 + a_0^2\cos^2\varphi_n}{a_0\sin\varphi_n}, \quad 2\psi = \varphi_m + (-1)^{m+n}\varphi_n\right)$$

We obtain analogous quantities dependent on $\mathbf{P}(\lambda)$ from (2.2) by replacing A_n by B_n. The remaining integrals in (1.5), (1.8), (1.9) are evaluated by using mechanical quadratures of the highest degree of accuracy /6/.

Taking account of (2.2) and (1.9) when substituting these integrals into (1.8), we find a system of linear algebraic equations to determine the constants A_n, B_n. Convergence of the collocation method for the particular kind of system (1.8) is proved in /2/.

The stress intensity coefficients on the endfaces of the reinforcement are determined by means of the formulas

$$\{k_M(\alpha_0), k_{\mathbf{p}}(\alpha_0)\} = \lim_{\varphi \to 0} \{M_\rho(\varphi), \mathbf{P}(\varphi)\} \sin \varphi = \frac{1}{N_0} \sum_{n=1}^{N_0} \{A_n, B_n\} (-1)^{n+1} \operatorname{ctg} \frac{\varphi_n}{2} \qquad (2.3)$$

$$\{k_M(-\alpha_0), k_{\mathbf{p}}(-\alpha_0)\} = \lim_{\alpha \to \pi} \{M_\rho(\varphi), \mathbf{P}(\varphi)\} \sin \varphi = \frac{1}{N_0} \sum_{n=1}^{N_0} \{A_n, B_n\} \operatorname{tg} \frac{\varphi_n}{2}$$

As an illustration, stresses on the hole contour are determined for a reinforcing rod of constant rectangular cross-section with the parameters

$$h_0/h = {}^4/_3, \ b_0/\rho_0 = 0.05, \ \omega_0 = 0.05, \ \alpha_0 = 5\pi/6 \qquad (2.4)$$

for $M_2 = 1$, $M_1 = 0$, $\beta_0 = 0$. All the calculations were performed on a computer for different values of N_0 to $N_0 = 64$. It should be noted that starting with $N_0 = 32$ the results are practically unchanged. Insignificant differences were observed only in immediate proximity to the endfaces of the reinforcing rod. The results of the computation are represented in Figs.2 and 3. The dashed, solid, and dash-dot lines correspond to values of E_0/E equal to 10, 100, 1000. Analysing the results obtained, we conclude that the influence of the reinforcing rod is insignificant for $E_0/E < 10$ while we obtain an absolutely rigid reinforcement for $E_0/E > 1000$.

3. Let two centrally-symmetric sections of the hole $\lambda \in [-\alpha_0, \alpha_0] \cup [\pi - \alpha_0', \pi + \alpha_0]$ be reinforced by rods with identical physical geometry characteristics. The external load on the plate is the same as in the preceding problem.

Taking into account $L \equiv [-\alpha_0, \alpha_0] \cup [\pi - \alpha_0, \pi + \alpha_0]$, and the dependences

$$\frac{1}{\pi} \int_L M_\rho(t)\,dt = \frac{2}{\pi} \int_{-\alpha_0}^{\alpha_0} M_\rho(t)\,dt \qquad (3.1)$$

$$\frac{1}{\pi} \int_L M_\rho(t) \operatorname{ctg} \frac{\lambda - t}{2}\,dt = \frac{2}{\pi} \int_{-\alpha_0}^{\alpha_0} M_\rho(t) \operatorname{ctg}(\lambda - t)\,dt$$
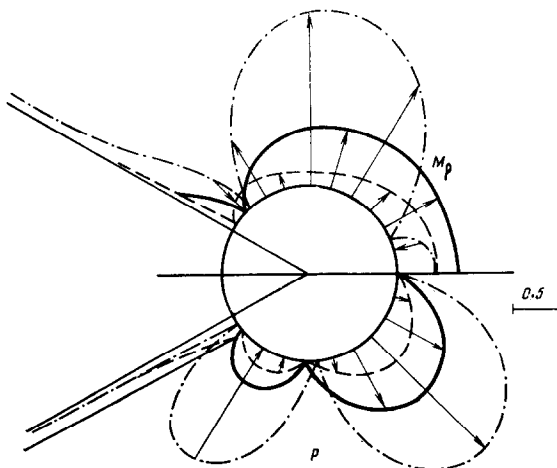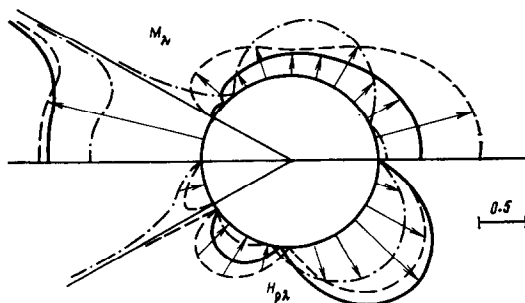
we find from (1.1), (1.3)

Fig.2

Fig.3

$$X = \frac{k}{3+v}[I_1(\lambda) + (1+v)M_\rho(\lambda) - X_0] \tag{3.2}$$

$$Y = -\frac{k}{3+v}[J_1(\lambda) + (1+v)\mathbf{P}(\lambda) - Y_0]$$

$$I_1(\lambda) = \frac{2}{\pi}\int_{-\alpha_\bullet}^{\alpha_\bullet} M_\rho(t)\,dt - \frac{2}{\pi}\int_{-\alpha_\bullet}^{\alpha_\bullet} \mathbf{P}(t)\operatorname{ctg}(\lambda - t)\,dt$$

$$J_1(\lambda) = \frac{2}{\pi}\int_{-\alpha_\bullet}^{\alpha_\bullet} \mathbf{P}(t)\,dt + \frac{2}{\pi}\int_{-\alpha_\bullet}^{\alpha_\circ} M_\rho(t)\operatorname{ctg}(\lambda - t)\,dt$$

Substituting (3.2) into the boundary conditions (1.6), we obtain a system of singular integral equations to determine the contact stresses

$$\delta[I_1(\lambda) + (1+v)M_\rho(\lambda) - X_0] = N(\lambda)\sin\lambda - R(\lambda)\cos\lambda \tag{3.3}$$

$$\delta\omega_0[J_1(\lambda) + (1+v)\mathbf{P}(\lambda) - Y_0] = -N(\lambda)\cos\lambda - R(\lambda)\sin\lambda$$

The equilibrium conditions (1.9) should be satisfied for each rod in addition to system (3.3).

We can determine the annular and twisting moments by means of (1.2).

The solution of system (3.3) has the form (2.1) for

$$\lambda = \operatorname{arctg} a_0\cos\varphi, \quad a_0 = \operatorname{tg}\alpha_0$$

We find the stress intensity coefficients by means of (2.3).

As an example, the stresses on a hole contour are determined for rods with the parameters (2.4) $(\alpha_0 = \pi/4)$ for $M_1 = 0$, $M_2 = 1$. The results of the computation are represented in Fig.4.
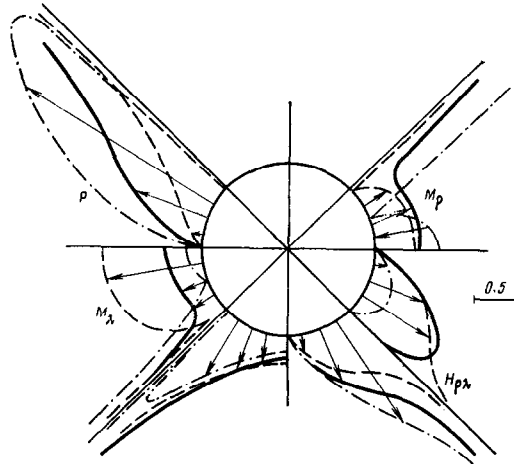

Fig.4

4. Let a curvilinear hole in an infinite plate be a regular polygon with rounded-off angles (hypotrochoid). Without loss of generality, we consider the line of juncture L between the plate and the reinforcing rod to be symmetrical about the Ox axis $(\beta_0 = 0)$.

The function

$$\omega(\xi) = \rho_0(\xi + m_0/\xi^{N-1}) \tag{4.1}$$

effects the conformal mapping of the domain occupied by the plate onto the exterior of the unit circle $|\xi| > 1$ in the complex ξ plane. Here N is the number of vertices of the polygon, and ρ_0 is the characteristic dimensions of the hole. For $N = 2$, $|m_0| < 1$ the function (4.1) effects conformal mapping of the exterior of an ellipse, for $N = 3$, $|m_0| < 1/2$ the exterior of a triangle with rounded-off angles, and for $N = 4$, $|m_0| < 1/3$ the exterior of a square with rounded-off angles on the exterior of the unit circle.

Separating the plate from the rod, we examine its equilibrium under the effect of the contact normal moment M_ρ and the tangential moment $\mathbf{P}$ applied along the line of juncture.

The contour conditions of the first and second boundary value problems for a plate have the form /1/

$$\varkappa\Phi_*{}^-(\sigma) + \Phi_*{}^+(\sigma) = -kf_*(\sigma) \tag{4.2}$$

$$\Phi_*{}^+(\sigma) - \Phi_*{}^-(\sigma) = \frac{k}{1+v}[M_\lambda - vM_\rho - i(1+v)H_{\rho\lambda}]\omega'(\sigma) \tag{4.3}$$

$$\Phi_* (\sigma) = \Phi (\sigma) \, \omega' (\bar{\sigma}), \quad \sigma = \exp (i\lambda)$$

$$f_* (\sigma) = f (\sigma) \, \omega' (\sigma) = (M_\rho + i\mathrm{P}) \, \omega' (\sigma)$$

The general solution of problem (4.2) bounded at "infinity", and satisfying the conditions defined by /1/ for $\xi = 0$ has the form

$$\Phi_*^- (\sigma) = \frac{k}{2\varkappa} \left[\frac{1}{\pi i} \int_\gamma \frac{f_* (\tau) \, d\tau}{\tau - \varsigma} - f_* (\sigma) \right] + \Gamma - \frac{1}{\varkappa} R (\sigma) \tag{4.4}$$

$$\Phi_*^+ (\sigma) = -\frac{k}{2} \left[\frac{1}{\pi i} \int_\gamma \frac{f_* (\tau) \, d\tau}{\tau - \sigma} + f_* (\sigma) \right] - \varkappa \Gamma + R (\sigma)$$

Here

$$R (\sigma) = [\Gamma' - {}^1\!/_2 \bar{b} m_0 \, (N - 2) \, (N - 3)] \, \sigma^{-2} + m_0 \, (N - 1) \, \Gamma \sigma^{-N} \tag{4.5}$$

$$m_0 \bar{b} + \varkappa b = - \Gamma' - \frac{k}{2\pi} \int_\gamma f_* (\sigma) \, \sigma^2 \, d\lambda$$

and γ is the image of the juncture line L in the mapping (4.1).

Taking account of (4.5) and substituting (4.4) into conditions (4.3), we find

$$X^* = \frac{k}{3 + \nu} [I^* (\lambda) + (1 + \nu) M_\rho^* (\lambda) - X_0] \tag{4.6}$$

$$Y^* = -\frac{k}{3 + \nu} [J^* (\lambda) + (1 + \nu) \mathrm{P}^* - Y_0^*]$$

$$M_\rho^* + i\mathrm{P}^* = (M_\rho + i\mathrm{P}) \, \omega' (\sigma), \quad X^* - iY^* = (X - iY) \, \omega' (\sigma) \tag{4.7}$$

$$X_0^* = X_0 - m_0 (N - 1) \frac{1 - \nu}{1 + \nu} (M_1 + M_2) \cos N\lambda + B \cos 2\lambda$$

$$Y_0^* = Y_0 + m_0 (N - 1) \frac{1 - \nu}{1 + \nu} (M_1 + M_2) \sin N\lambda - B \sin 2\lambda$$

$$B = \frac{m_0 (N - 2)(N - 3)}{\varkappa + m_0} \left[(M_1 - M_2) - \frac{1}{\pi} \int_\gamma M_\rho^* \cos 2t - \mathrm{P}^* \sin 2t \Big) dt \right]$$

Outwardly expressions (4.6) agree with the correspondingly quantities (1.4) for a circular hole. If they were known, the components of the state of stress and strain on the outline of a curvilinear hole would be determined by means of the formulas

$$M_\rho = \frac{\alpha M_\rho^* + \beta \mathrm{P}^*}{|\omega' (\sigma)|^2}, \quad \mathrm{P} = \frac{\alpha \mathrm{P}^* - \beta M_\rho^*}{|\omega' (\sigma)|^2} \tag{4.8}$$

$$X = \frac{\alpha X^* - \beta Y^*}{|\omega' (\sigma)|^2}, \quad Y = \frac{\alpha Y^* + \beta X^*}{|\omega' (\sigma)|^2}$$

$$M_\lambda = \nu M_\rho - \frac{1 + \nu}{k} X, \quad H_{\rho\lambda} = - k^{-1} Y, \quad \alpha + i\beta = \omega' (\sigma)$$

We find the boundary conditions of the plate and rod connection by projecting the external load acting on one side of the section s on the tangent and normal

$$L_\rho - iL_\lambda = - i \int_s^{s_0} (M_\rho + i\mathrm{P}) \exp [i (\theta_1 - \theta)] \, ds_1 \tag{4.9}$$

$$\exp (i\theta_1) = t_1 \frac{\omega' (t_1)}{|\omega' (t_1)|}, \quad \exp (i\theta) = \sigma \frac{\omega' (\sigma)}{|\omega' (\sigma)|}$$

Here s_0 is the rod endface section, and θ, θ_1 are the angles between the normals to the hold outline and the Ox axis in the sections s and s_1 respectively.

Substituting their expressions (1.7), taking (4.8) into account for L_ρ and L_λ into (4.9), and separating real and imaginary parts, we find the final version of the boundary condition

$$X^* = \frac{1}{A |\omega' (\sigma)|} \left[- Q^* \Big(\alpha^2 + \frac{\beta^2}{\omega_0} \Big) + R^* \alpha\beta \Big(1 - \frac{1}{\omega_0} \Big) \right] \tag{4.10}$$

$$Y^* = \frac{1}{A |\omega' (\sigma)|} \left[Q^* \alpha\beta \Big(1 - \frac{1}{\omega_0} \Big) - R^* \Big(\beta^2 + \frac{\alpha^2}{\omega_0} \Big) \right]$$

$$R^* = \int_\lambda^{\alpha_*} [M_\rho^* \cos (\lambda - t) - \mathrm{P}^* \sin (t - \lambda)] \, dt$$

$$Q^* = \int_\lambda^{\alpha_*} [M_\rho^* \sin (t - \lambda) + \mathrm{P}^* \cos (t - \lambda)] \, dt$$

Taking (4.6) into account we reduce relationships (4.10) to a system of two singular integral equations to determine the quantities $M_\rho{}^*$ and P^*

$$I^*(\lambda) + (1 + \nu) M_\rho{}^*(\lambda) - X_0{}^* +$$
$$\delta^{-1}\left[Q^*\left(\alpha^2 + \frac{\beta^2}{\omega_0}\right) - R^*\alpha\beta\left(1 - \frac{1}{\omega_0}\right)\right] = 0 \qquad (4.11)$$
$$J^*(\lambda) + (1 + \nu) P^*(\lambda) - Y_0{}^* +$$
$$\delta^{-1}\left[Q^*\alpha\beta\left(1 - \frac{1}{\omega_0}\right) - R^*\left(\beta^2 + \frac{\alpha^2}{\omega_0}\right)\right] = 0$$

The equilibrium conditions for a rod as a rigid whole

$$\int_{-\alpha_\bullet}^{\alpha_\bullet} (M_\rho{}^* \cos t - \mathbf{P}^* \sin t)\, dt = 0 \qquad (4.12)$$
$$\int_{-\alpha_\bullet}^{\alpha_\bullet} (M_\rho{}^* \sin t + \mathbf{P}^* \cos t)\, dt = 0$$

should be satisfied as well as system (4.11).

System (4.11) has the same structure as the corresponding system (1.8) for a circular hole. This means that the method of boundary collocation can be applied successfully for its approximate solution.

If the solution of system (4.11) that satisfies the equilibrium conditions (4.12) is known, then the components of the state of stress and strain on the contour of a curvilinear hole can be determined by means of (4.8).

Setting $|\omega'(\sigma)| = \rho_0$ $(\alpha = 1, \beta = 0)$, into (4.11), (4.12), we obtain the well-known relations (1.8), (1.9) for a plate with a partially reinforced circular hole.

The proposed method of solution can be applied to determine the state of stress in composite, piecewise-homogeneous plates with curvilinear stiffener ribs.

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Translated by M.D.F.